

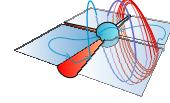


# **X WORKSHOP**

on Dynamical Systems

# **2019**

# **Posters**



## Compact leaves of one dimensional principal foliations associated to a plane field in three dimension

Gomes, Alacyr J.

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*

*alacyr@ufg.br*

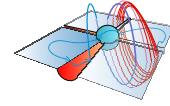
**Abstract:** In this work we will consider a plane distribution  $\Delta\eta$  defined by a regular vectors field  $\eta : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ . The principal lines associated to  $\Delta\eta$  are the integral curves defined by the implicit differential equation

$$2[D\eta(dr), dr, \eta] + \langle \text{rot}(\eta), \eta \rangle \cdot \langle dr, dr \rangle = 0, \quad \langle \eta, dr \rangle = 0,$$

which was established by Y. Aminov (2000).

It will be shown that generically the compact leaves of these foliations are hyperbolic, making use of results from geometric control theory.

In the case where  $\Delta\eta$  is integrable these foliations are exactly the principal curvature lines of a one parameter family of surfaces in  $\mathbb{R}^3$ , a classical subject of differential geometry of surfaces and was introduced by G. Monge (1796). The qualitative theory and global aspects of principal lines were initiated by C. Gutierrez and J. Sotomayor (1982).



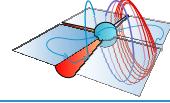
## Dinâmica de partículas autopropelidas em fluxos

Jara Grados, Alfredo

*Universidade Federal do ABC*

*alfredo.jara@ufabc.edu.br*

**Resumo:** Estudamos um modelo de partículas autopropelidas em um fluxo bidimensional estacionário correspondente a células convectivas com barreiras de transporte. Essas partículas têm forma de esferoide prolato e sua orientação satisfaz as equações de Jeffery. Consideramos que as partículas têm uma velocidade de autopropulsão diferente de zero quando sua orientação está próxima a uma direção predeterminada. Obtemos um mapa Poincaré sobre o plano de discontinuidade do campo de velocidades e apresentamos os resultados obtidos. Utilizamos métodos numéricos e analíticos para abordar os objetivos.



## Estabilidade estrutural em campos de vetores suaves por partes definidos na esfera $\mathbb{S}^2$

Ana Livia Rodero

*Instituto de Biociências, Letras e Ciências Exatas - Universidade Estadual Paulista*  
*analivia.alr@gmail.com*

**Resumo:** Neste trabalho exibimos uma classificação do conjunto dos campos de vetores suaves por partes definidos na esfera  $\mathbb{S}^2 \subset \mathbb{R}^3$ , que tem o equador  $\Sigma = \{(x, y, z) \in \mathbb{S}^2; z = 0\}$  como região de descontinuidade, que são estruturalmente estáveis.

Mais precisamente, consideramos  $\mathbb{S}^2 \subset \mathbb{R}^3$ ,  $F : \mathbb{S}^2 \rightarrow \mathbb{R}$  tal que  $F(x, y, z) = z$ ,  $\Sigma^+ = F^{-1}(0, \infty)$  e  $\Sigma^- = F^{-1}(-\infty, 0)$ . Sejam  $\chi^r = \chi^r(\mathbb{S}^2)$  o espaço dos campos vetoriais de classe  $C^r$ ,  $r \geq 1$ , definidos em  $\mathbb{S}^2$  e, seguindo a convenção de Filippov, considere  $\Omega^r = \Omega^r(\mathbb{S}^2, F)$  o espaço dos campos vetoriais  $Z$  em  $\mathbb{S}^2$  definidos por:

$$Z(x) = \begin{cases} X(x), F(x) \geq 0, \\ Y(x), F(x) \leq 0 \end{cases} \text{ onde } X, Y \in \chi^r.$$

Considere  $\Sigma_0^{S^2} \subset \Omega(\mathbb{S}^2, F)$ , definido da seguinte forma:

$$\Sigma_0^{S^2} = \left\{ \begin{array}{l} Z = (X, Y) \in \Omega(\mathbb{S}^2, F) \text{ tal que,} \\ \text{(a) Todos os equilíbrios de } Z \text{ são hiperbólicos e não estão em } \Sigma; \\ \text{(b) Todas as órbitas periódicas são hiperbólicas e não tocam } \Sigma; \\ \text{(c) Todas as } \Sigma-\text{singularidades são elementares;} \\ \text{(d) Todas as poli-trajetórias fechadas de } Z \text{ são elementares;} \\ \text{(e) } Z \text{ não possui conexão de separatrizes nem conexões de tangências.} \end{array} \right\}$$

**Teorema.**  $Z \in \Omega(\mathbb{S}^2, F)$  é estruturalmente estável se, e somente se,  $Z \in \Sigma_0^{S^2}$ . Além disso,  $\Sigma_0^{S^2}$  é aberto.

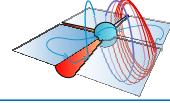
Nosso próximo objetivo é caracterizar o conjunto do campos de codimensão um em  $\Omega(\mathbb{S}^2, F)$ .

Este é um trabalho em colaboração com Claudio Aguinaldo Buzzi (IBILCE/UNESP). Este trabalho é financiado pela FAPESP. Número do processo 2017/08779-8.

## Referências

- [1] Guardia, M., Seara, T., Teixeira, M. A., *Generic bifurcation of low codimension of planar filippov systems*. J. Differ. Equ. 250(4), 1957-2023 (2011)
- [2] Peixoto, M. C., Peixoto, M. *Structural Stability in the plane with enlarged boundary conditions*. An. Acad. Bras. Cie., 31, 135 (1959)

- [3] Sotomayor, J., Machado, A. L. F. *Structurally stable discontinuous vector fields in the plane*. Qual. Theory Dyn. Syst. 3, pp. 227-250 (2002)
- [4] Kuznetsov, Y. A., Rinaldi, S. Gragnani, A. *One-parameter bifurcations in planar filippov systems*. Int. J. Bifurc. Chaos 13(8), 2157-2188 (2003)



# Dinâmica global de certos sistemas lineares descontínuos no plano separados pela união de duas semirretas

Ana Maria Alves da Silva

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*

*anamarias.ufg@gmail.com*

**Resumo:** Nos últimos anos houve um interesse considerável no estudo dos sistemas lineares por partes. Existe um interesse especial em estudar a existência, o número e a distribuição dos ciclos limites em sistemas lineares por partes do plano. Em [1] os autores demonstram a não existência de ciclos limites para sistemas lineares contínuos com centros no plano com duas zonas e com três zonas. Os autores demonstram também que para sistemas lineares descontínuos com centros no plano com duas zonas não há ciclos limite mas para o caso de 3 zonas há e a cota é realizável, além disso, os autores propõem uma forma normal para sistemas lineares com centros.

Consideremos os seguintes campos:

$$X(x, y) = \left( -bx - \frac{4b^2 + w^2}{4a}y + d, ax + by + c \right) \quad a > 0, \quad w > 0$$

$$Y(x, y) = \left( -Bx - \frac{4B^2 + W^2}{4A}y + D, Ax + By + C \right) \quad A > 0, \quad W > 0.$$

Os campos  $X$  e  $Y$  têm centros lineares tomados a partir da forma normal proposta em [1]. Tome  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  dada por  $f(x, y) = y - |x|$  e defina  $X^+ = X|_{\Sigma^+}$  e  $X^- = Y|_{\Sigma^-}$  onde  $\Sigma = f^{-1}(0)$ ,  $\Sigma^+ = \{(x, y) \in \mathbb{R}^2; f(x, y) \geq 0\}$  e  $\Sigma^- = \{(x, y) \in \mathbb{R}^2; f(x, y) \leq 0\}$ . Defina agora o seguinte sistema linear suave por partes no plano:

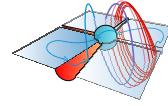
$$Z(x, y) = \begin{cases} X^+(x, y), & (x, y) \in \Sigma^+ \\ X^-(x, y), & (x, y) \in \Sigma^- \end{cases}. \quad (1)$$

Em nosso trabalho, estamos interessados em estudar a dinâmica (tangências; regiões de deslize, escape, costura, etc.) e a existência de ciclos limites para o sistema definido em (1).

Este é um trabalho em colaboração com Rodrigo Donizete Euzébio (IME/UFG).

## Referências

- [1] Llibre, Jaume; Marco A. Teixeira. *Piecewise linear differential systems with only centers can create limit cycles?* Nonlinear Dynamics, v. 91, p. 249-256, 2018.



## Integrability of quadratic systems with invariant hyperbolas

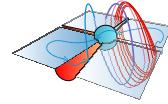
Ana Maria Travaglini

Instituto de Ciências Matemáticas e de Computação - Universidade de São Paulo  
[anamtr@usp.br](mailto:anamtr@usp.br)

**Abstract:** The goal of this work is to present the investigation in development about integrability of planar quadratic differential systems in the whole class of non-degenerate planar quadratic differential systems possessing at least one invariant hyperbola (QSH). Such class was investigated in [1] where the authors classify it according to its geometric properties encoded in the configurations of invariant hyperbolas and invariant straight lines which these systems possess. In this poster we shall present results about Darboux and Liouvillian integrability of such systems and investigated geometric properties of them.

## References

- [1] Oliveira, R. D. S., Rezende, A. C., Schlomiuk, D., Vulpe, N. *Geometric and Algebraic Classification of Quadratic Differential Systems with Invariant Hyperbolas*. Electronic Journal of Differential Equations 2017, 295 (2017), 1-112.



## Conjugations of piecewise vector fields and shift spaces

Andre do Amaral Antunes

*Instituto de Biociências, Letras e Ciências Exatas - Universidade Estadual Paulista  
andre.antunes@unesp.br*

**Abstract:** Chaos in Piecewise Vector Fields (PSVF, for short) was already discussed in some papers (see [1], [2]). In this work, we will construct a conjugation between some PSVF and two sided shift spaces with any amount of symbols.

The main idea behind this construction is to divide the invariant set where the dynamics is chaotic into compartments and then assign to each compartment one symbol of our shift space. This yields a function of any possible orbit of the vector field to the shift space.

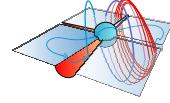
From this relation, we want to translate properties of shift spaces (e.g. entropy), to PSVF, meaning that, for example, one could answer the question of whether one system is "more"chaotic than another.

Joint work with Tiago de Carvalho (USP - Ribeirão Preto).

This work is funded by FAPESP (Process number 2017/18255-6).

## References

- [1] Colombo, A. and Jeffrey, M. R. *Nondeterministic Chaos, and the Two-fold Singularity in Piecewise Smooth Flows*. SIAM Journal on Applied Dynamical Systems 10, 2 (2011) 423-451.
- [2] Buzzi, C. A., Carvalho, T. and Euzébio, R. D. *Chaotic planar piecewise smooth vector fields with non-trivial minimal sets*. Ergodic Theory and Dynamical Systems 36, 2 (2016) 458-469.

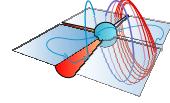


## Symbolic dynamics of piecewise contractions

Benito Pires

Universidade de São Paulo - Ribeirão Preto  
[benito@usp.br](mailto:benito@usp.br)

**Abstract:** A map  $f : [0, 1] \rightarrow [0, 1]$  is a *piecewise contraction* if locally  $f$  contracts distance, i.e., if there exist  $0 < \lambda < 1$  and a partition of  $[0, 1]$  into intervals  $I_1, I_2, \dots, I_n$  such that  $|f(x) - f(y)| \leq \lambda|x - y|$  for all  $x, y \in I_i$  ( $1 \leq i \leq n$ ). Piecewise contractions describe the dynamics of many systems such as traffic control systems, queueing systems, outer billiards and Cherry flows. Here I am interested in the symbolic dynamics of such maps. More precisely, we say that an infinite word  $i_0i_1i_2\dots$  over the alphabet  $\mathcal{A} = \{1, 2, \dots, n\}$  is the *natural coding* of  $x \in [0, 1]$  if  $f^k(x) \in I_{i_k}$  for all  $k \geq 0$ . The aim of this talk is to provide a complete classification of the words that appear as natural codings of injective piecewise contractions.



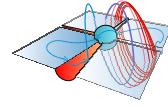
# Inflection points on hyperbolic tori of $\mathbb{S}^3$

B. Freitas

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*  
*brfmat@gmail.com*

**Abstract:** Families of hyperbolic tori in  $\mathbb{S}^3$  (the asymptotic lines are globally defined) without double inflection points is provided. More precisely, a small deformation of the Clifford torus parametrized by asymptotic lines is analyzed and it is described by the set of inflections of the two families of asymptotic lines  $\mathcal{A}_1$  and  $\mathcal{A}_2$ . Denote by  $\mathcal{I}_i$  the set of inflections of the asymptotic lines of the associated asymptotic foliation  $\mathcal{A}_i$ , also called flecnodal set. The intersection  $\mathcal{I}_1 \cap \mathcal{I}_2$  is called the set of double inflections. It is shown that by an appropriated deformation of the Clifford torus the set  $\mathcal{I}_1 \cap \mathcal{I}_2$  is empty for the deformed surface. This gives a negative answer to a problem formulated by S. Tabachnikov and V. Ovsienko [Hyperbolic Carathéodory Conjecture, Proc. of the Steklo Inst. of Math. **258** (2007), p. 178-193] in the context of spherical surfaces.

Joint work with R. Garcia (IME/UFG).



## Direction transition matrix

Cícero Rumão G. de Sousa Jr.

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*  
*cicero.rumao@hotmail.com*

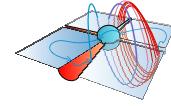
**Abstract:** We present a transition matrix given by the directional transition matrix that covers the flow defined Conley isomorphism. We will show some examples and a computational application using Mathematica software.

The directional transition matrix was developed in [3] and [4], the advantage of this more general approach is that it allows us to detect larger families of bifurcation orbits in continuation than those that are detected by the singular and topological transition matrices.

The work will be based in [1] and [2].

## References

- [1] Robert Franzosa, Ketty A. de Rezende, and Ewerton R. Vieira, *Generalized topological transition matrix*, Topol. Methods Nonlinear Anal. **48** (2016), no. 1, 183-212. MR 3561428
- [2] Robert Franzosa and Ewerton R. Vieira, *Transition Matrix Theory*, Trans. Amer. Math. Soc. **369** (2017), no. 11, 7737-7764. MR 3695843
- [3] Tomás Gedeon, Hiroshi Kokubu, Konstantin Mischaikow, Hiroe Oka, and James F. Reineck, *The Conley index for fast-slow systems. I. One-dimensional slow variable*, J. Dynam. Differential Equations 11 (1999) , no. 3, 427-470. MR 1693854.
- [4] Hiroshi Kokubu, Konstantin Mischaikow, and Hiroe Oka, *Directional transition matrix*, Conley index theory (Warsaw, 1997), Banach Center Publ., vol. 47, Polish Acad. Sci., Warsaw, 1999, pp. 133-144. MR 1692367.



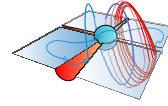
## High order Melnikov analysis in nonsmooth differential systems with nonlinear switching manifold

Dayane R. Cruz

*Instituto de Matemática, Estatística e Computação Científica  
Universidade Estadual de Campinas*

*dayanemat29@gmail.com*

**Abstract:** The Averaging Theory states that periodic solutions of a perturbed system bifurcating a continuum of periodic solutions can be detected by the “Averaged” functions of the system. This holds for continuous systems, however it was noticed that the same does not occur for discontinuous systems. In “Melnikov analysis in nonsmooth differential systems with nonlinear switching manifold” article written by Dr. Jéfferson L. R. Bastos, Dr. Claudio Buzzi, Dr. Jaume Llibre, and Dr. Douglas D. Novaes, it was demonstrated that the functions of bifurcation, also called of Melnikov’s functions, which control the bifurcation of periodic solutions differ from the “averaged” ones by an increment that depends on the “discontinuity jump” and the geometry of the switching manifold. The purpose of this work is to generalize the previous study for higher order perturbations.



## Conley Index Theory for trajectories periodic with sliding region

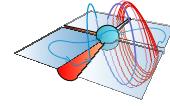
Ewerton Rocha Vieira

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*  
*ewerton@ufg.br*

**Abstract:** We present results of Conley Index Theory that guarantee existence of periodic trajectories with sliding region, see [1, 2, 3, 4].

## References

- [1] Conley, C. *Isolated Invariant Sets and the Morse Index*, CBMS Regional Conference Series in Mathematics, 38. American Mathematical Society, Providence, R.I., (1978). iii+89 pp.
- [2] Franzosa, R.; de Rezende, K. A.; Vieira, E. R. *Generalized Topological Transition Matrix*, Topol. Methods Nonlinear Anal. 48 (2016), no. 1, 183–212.
- [3] Franzosa, R.; Vieira, E.R. *Transition matrix theory*. Transactions of the American Mathematical Society, <http://www.ams.org/journals/tran/earlyview/> (2017).
- [4] McCord, C.; Mischaikow, K.; Mrozek, M. *Zeta Functions, Periodic Trajectories, and the Conley Index*, Journal of Differential Equations, 121, 258-292 (1995).



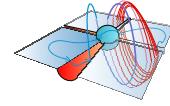
## Inseparable leaves in the plane

Filipe Balduino Pires Fernandes

*Departamento de Matemática - Universidade Federal de São Carlos*

*filipefernandes@dm.ufscar.br*

**Abstract:** We provide a new lower bound for the maximum number of inseparable leaves a chordal system in the plane of degree  $n$  can have by using the blow-up technique in a different way.



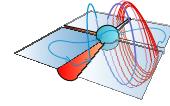
# Regularization of Filippov systems near regular-tangential singularities and tangential polycycles

Gabriel Alexis Rondón Vielma

*Instituto de Matemática, Estatística e Computação Científica  
Universidade Estadual de Campinas*

*garv202020@gmail.com*

**Abstract:** Understanding how tangential singularities evolves under regularization processes was one of the first problem concerning regularization of Filippov systems. Indeed, in the earlier work of Sotomayor and Teixeira, it is proved that around a regular-fold singularity of a Filippov system  $Z$ , the regularized system  $Z$  possesses no singularities. Recently, some works got deeper results by studying the corresponding slow-fast problems. In some articles asymptotic methods were used to study  $C_n$ -regularizations of generic regular-fold singularities and fold-fold singularities, respectively. In other articles, the Blow-up method was adapted to study  $C_n$ -regularizations of fold-fold singularities and an analytic regularization of a regular-fold singularity, respectively. In this work, we are interested in  $C_n$ -regularizations of regular-tangential singularities through blow-up methods. Our main result extend for degenerate regular-tangential singularities the results obtained for regular-fold singularities. The main difference between our problem and the problem addressed in the other works is that a regular-fold singularity admits a normal form which simplify a lot the study. Here, we have to deal with higher order terms. We are also interested in  $C_n$ -regularizations of Filippov systems containing regular-tangential polycycles.



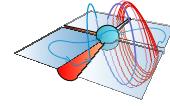
# A framework to study piecewise smooth dynamical systems in the presence of a singular switching manifold

Guilherme Tavares da Silva

*Instituto de Matemática, Estatística e Computação Científica  
Universidade Estadual de Campinas*

*ra119509@ime.unicamp.br*

**Abstract:** The theory of piecewise smooth dynamical systems, in particular, Filippov systems, provides a natural framework for modeling various phenomena in the nature and has had a fast development in the last decades. However, most of the studies in this area are directed to Filippov systems with regular switching manifolds, where the Filippov dynamics is well defined. In this work, we study a 3D cross-like singular configuration of the switching manifold, where the Filippov dynamics isn't well defined. More precisely, we propose a framework that, through blow-ups and slow-fast theory, extends Filippov dynamics to this singular configuration (and possibly others). This framework has been applied to the piecewise linear case and, until now, we have successfully unfolded the dynamics of the constant subcase. A complete unfold of the general linear case is expected in a near future. Joint work with Ricardo Miranda Martins.



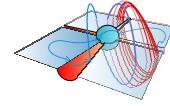
## Rational first integrals of the Lienard equations: the solution to the Poincaré problem for the Lienard equation

Jarne Donizetti

*Instituto de Biociências, Letras e Ciências Exatas - Universidade Estadual Paulista*  
*jarnedonizetti@yahoo.com.br*

**Abstract:** Poincaré in 1891 asked about the necessary and sufficient conditions in order to characterize when a polynomial differential system in the plane has a rational first integral. Here we solve this question for the class of Liénard differential equations  $\ddot{x} + f(x)\dot{x} + x = 0$ , being  $f(x)$  a polynomial of arbitrary degree. As far as we know it is the first time that all rational first integrals of a relevant class of polynomial differential equations of arbitrary degree has been classified.

Joint work with Jaume Llibre (UAB) and Claudio Pessoa (IBILICE/UNESP).

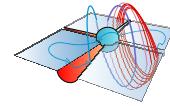


## Fibrations and global injectivity of local homeomorphisms

Jean Venato Santos

*Universidade Federal de Uberlândia*  
*jvenatos@ufu.br*

**Abstract:** Given  $X$  a path connected space and  $g: X \rightarrow \mathbb{R}$  a local fibration on its image, we prove that  $g$  satisfies a kind of deformation and consequently we obtain the path connectedness of its level sets. Then we provide global injectivity and invertibility theorems for local homeomorphisms  $f: X \rightarrow \mathbb{R}^n$ . These generalize known analytical results such as those given by Balreira and by Silva and Teixeira.



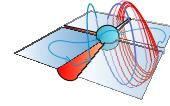
## Uma condição suficiente para recorrência na T-singularidade

Jeferson Cassiano

*Universidade Federal do ABC*

*jeferson.cassiano@ufabc.edu.br*

**Resumo:** Neste trabalho apresentamos algumas condições suficientes sobre a forma normal do campo e da aplicação do primeiro retorno na vizinhança da T-singularidade.



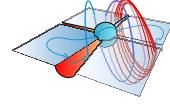
## Sobre conjuntos limite para uma classe de sistemas de Filippov possuindo movimento deslizante

Joaby Jucá

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*

*joabymat@gmail.com*

**Resumo:** Neste trabalho, a estrutura global de trajetórias de uma classe sistemas de Filippov no plano, que são confinadas em um compacto são estudadas. É assumido que o sistema de Filippov é definido por dois campos de vetores  $C^r$  cujas trajetórias são separadas por uma curva suave  $\Sigma$ , cada um possuindo no máximo um ponto de tangência com  $\Sigma$ . O comportamento das trajetórias globais é completamente analizado e seus conjuntos limite são classificados. Além disso, as trajetórias podem comutar com  $\Sigma$  infinitas vezes, inclusive visitando as regiões de deslise ou escape. Consequentemente, apresentamos uma versão do aclamado Teorema de Poincaré-Bendixson sem evitar movimento deslizante. Nesta direção alguns conjuntos (limite) apresentando movimento deslizante ou mesmo interior não-vazio são obtidos. No último caso, provamos a existência de caos não determinístico. Algumas características importantes das trajetórias são distinguidas analizando condições simples em termos dos pontos de tangência, como condições suficientes para a existência de ciclos limite deslizantes. Finalmente, exemplos de classes importantes de sistemas de Filippov que se ajustam as nossas hipóteses, como os lineares por partes, são apresentados, e vemos que pode ocorrer caos mesmo neste caso.



# Crossing limit cycles for a class of piecewise linear differential centers separated by a conic

J. C. Medrado

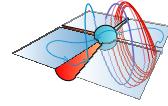
*Instituto de Matemática e Estatística - Universidade Federal de Goiás*  
*medrado@ufg.br*

**Abstract:** These last years the study of the version of Hilbert's 16th problem for piecewise linear differential systems in the plane, has increased strongly and there are many papers studying the maximum number of crossing limit cycles when the differential system is defined in two zones separated by a straight line, in particular in [1, 2] it was proved that piecewise linear differential centers separated by a straight line have no crossing limit cycles, but in the papers [3, 4] it was shown that the maximum number of crossing limit cycles of piecewise linear differential centers, can change depending of the shape of the discontinuity curve. In this work we study the maximum number of crossing limit cycles of piecewise linear differential centers separated by a conic.

Joint work with J. Jimenez (IME/UFG) and J. Llibre (UAB).

## References

- [1] J. Llibre, D.D. Novaes and M. A. Teixeira, *Maximum number of limit cycles for certain piecewise linear dynamical systems*, Nonlinear Dyn. **82** (2015), 1159–1175.
- [2] J. Llibre and M. A. Teixeira, *Piecewise linear differential systems with only centers can create limit cycles?* Nonlinear Dyn. **91** (2018), 249–255.
- [3] J. Llibre and M. A. Teixeira, *Limit cycles in Filippov systems having a circle as switching manifold*, preprint, (2018).
- [4] J. Llibre and X. Zhang, *Limit cycles for discontinuous planar piecewise linear differential systems separated by an algebraic curve*, to appear in Int. J. Bifurcation and Chaos, (2019).

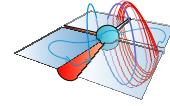


## Existence of $C^k$ -invariant foliations for Lorenz-type maps

Jose Vidarte

*Instituto de Matemática e Computação - Universidade Federal de Itajubá*  
*vidarte2283@gmail.com*

**Abstract:** In this poster, under conditions similar to those in Shashkov and Shil'nikov (1994) we show that a  $C^{k+1}$  Lorenz-type map  $T$  has a  $C^k$  codimension one foliation which is invariant under the action of  $T$ .



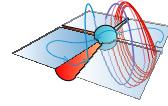
## Topological transitivity of Filippov systems

Régis Varão

*Instituto de Matemática, Estatística e Computação Científica  
Universidade Estadual de Campinas  
regisvarao@ime.unicamp.br*

**Abstract:** We deal with Filippov systems defined on a 2-dimensional manifold. We analyse the concept of topological transitivity for these systems. We obtain similar results for the context of uniqueness of flows, but once sliding or escaping regions are present we are able to say much more things. Transitivity in this case is much more related to chaos (devaney's sense).

Joint work with R. Euzébio.



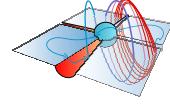
## Shilnikov connections arising from boundary equilibrium bifurcations

Kamila da S. Andrade

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*

*kamila.andrade@ufg.br*

**Abstract:** Generally speaking, a Shilnikov connection is a homoclinic connection through a hyperbolic saddle-focus equilibrium. In the nonsmooth context, it is possible to obtain a Shilnikov like connection through a pseudo saddle-focus equilibrium in the sliding region. In this work, we provide some models of nonsmooth systems presenting Shilnikov connections arising from boundary equilibrium bifurcations. Joint work with D. D. Novaes (UNICAMP).



# Hybrid bifurcation of limit cycles in quadratic piecewise system

Leonardo P. C. da Cruz

*Universidade Federal de São Carlos*

*leonardopereira@dm.ufscar.br*

**Abstract:** Consider the piecewise polynomial vector field

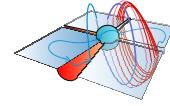
$$Z^\pm = \begin{cases} \dot{x} = -b^\pm y \pm a^\pm x \pm (b^\pm)^2 y^2 \mp (4/3(a^\pm)^2 + 1/3(b^\pm)^2 + 2) x^2, \\ \dot{y} = b^\pm x \pm a^\pm y - a^\pm b^\pm y^2 \mp ((a^\pm)^2 + (b^\pm)^2 + 1) xy + \\ \quad + ((5(a^\pm)^2 - 10(b^\pm)^2 + 9) a^\pm) x^2 / (15b^\pm), \end{cases} \text{ if } (x, y) \in \Sigma^\pm,$$

with  $\Sigma^\pm = \{(x, y) : \pm h(x, y) > 0\}$ , where  $h(x, y) = y - \tan(\epsilon)x$ . System (1) has two weak foci of order three in  $(0, \pm 1/b^\pm)$  and a focus at the origin. We study the number of periodic orbits that bifurcate from the three foci using the Lyapunov constants theory. This normal form is an adaptation of the result in [1] for piecewise systems. For this extension, we find a point in the parameter space corresponding to a hybrid configuration of limit cycles  $(3, 5, 3)$ .

Joint work with Alex C. Rezende and Joan Torregrosa.

## Referências

- [1] S.L. Shi, *A concrete example of the existence of four limit cycles for plane quadratic systems*. Sci. Sinica. **23** pp. 153–158 (1980).



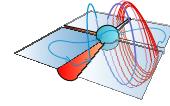
## Análise qualitativa local do sistema de Rossler

Luan Lima da Silva

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*  
*luan.lima.lemann@hotmail.com*

**Resumo:** Neste trabalho serão apresentados estudos matemáticos de caráter qualitativo e numérico do sistema de Rossler, os quais se remetem a descrever a dinâmica do sistema em inúmeros aspectos. Será feita uma abordagem analítica do sistema, isto por intermédio de resultados clássicos da teoria qualitativa de sistemas dinâmicos, em destaque o teorema de Grobman-Hartman, seguido também de uma análise numérica do sistema, a qual se remete a estudar, sob determinadas condições, pontos críticos do sistema e a sua linearização. Por fim serão considerados alguns problemas práticos em que o sistema pode ser empregado.

Este é um trabalho em colaboração com Caio Augusto Santos Magalhães (IME/UFG).



## Bifurcação de ciclos limites a partir de centros em variedades centrais de campos de vetores em $\mathbb{R}^3$

Lucas Queiroz Arakaki

*Instituto de Biociências, Letras e Ciências Exatas - Universidade Estadual Paulista*  
*lucas.queiroz@unesp.br*

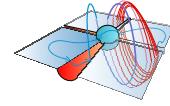
**Resumo:** Neste trabalho, apresentamos um método de estimativa da ciclicidade para sistemas tridimensionais que têm um centro na origem sobre a variedade central. Mais precisamente, consideramos perturbações do sistema

$$\begin{cases} \dot{u} = -v + P(u, v, w), \\ \dot{v} = u + Q(u, v, w), \\ \dot{w} = -\mu w + R(u, v, w), \end{cases}$$

que mantém a singularidade na origem e a propriedade que os autovalores da parte linear do campo vetorial correspondente sejam dois imaginários puros e um real não nulo.

Para os sistemas considerados, podemos definir os coeficientes focais  $g_{kk0}$  e os coeficientes de Lyapunov  $L_k$  que dependem dos parâmetros de perturbação  $\lambda \in \Lambda \subset \mathbb{R}^n$ . Estes coeficientes caracterizam a *variedade de Bautin*, o conjunto dos parâmetros para os quais o sistema tem um centro na origem sobre a variedade central. Enunciámos o teorema que exibe a conexão entre a independência das partes lineares dos coeficientes (tanto os de Lyapunov como os focais) e o número de ciclos limites de pequena amplitude que bifurcam a partir de um ponto da variedade de Bautin. Exemplificámos, aplicando o teorema para o sistema de Moon-Rand.

Este é um trabalho em colaboração com Claudio Gomes Pessoa (IBILCE/UNESP).



# Global phase portraits of the quadratic polynomial Liénard differential systems

Luci Any Roberto

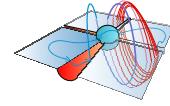
*Instituto de Biociências, Letras e Ciências Exatas - Universidade Estadual Paulista*  
*luci.roberto@unesp.br*

**Abstract:** The quadratic polynomial differential systems and their applications have been studied intensively these last thirty years. In particular, the polynomial Liénard differential systems and their applications also have been analyzed by many authors these recent years. Up to now the phase portraits in the Poincaré disc of the quadratic polynomial Liénard differential systems have not been studied, their study is the goal of this work. More precisely, we classify the different topological phase portraits in the Poincaré disc of the systems

$$\dot{x} = y, \quad \dot{y} = (ax + b)y + cx^2 + dx + e,$$

where  $(x, y) \in \mathbb{R}^2$  are the variables and  $a, b, c, d, e$  are real parameters.

Joint work with M. Gouveia (IBILCE-UNESP) and J. Llibre (UAB-Barcelona).



## Critical curves for the total normal curvature on surfaces

Luis Felipe Narvaez Plaza

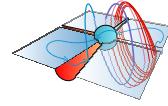
*Instituto de Matemática e Estatística - Universidade Federal de Goiás*  
*pipenarvaez91@gmail.com*

**Abstract:** Examples of curves derived from the calculation of variations in geometry have been extensively studied. Some curves initially defined with geometric properties can be characterized as a solution of variational problems. A known example of such curves is the geodetic curve.

Our aim in this work is to find the curves for which the total normal curvature

$$K_n = \int_p^q k_n ds,$$

is an extremal and to study the dynamical and geometric properties of those curves.



## On the formation of nested invariant tori and hidden chaotic attractor in the Sprott A system

Marcelo Messias

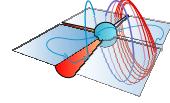
Universidade Estadual Paulista - Presidente Prudente

[marcelo@fct.unesp.br](mailto:marcelo@fct.unesp.br)

**Abstract:** We consider the well-known Sprott A system, which is a particular case of the Nosé-Hoover oscillator, related to problems of molecular dynamics. It is a three-dimensional quadratic differential system depending on a real parameter  $a$  and, for  $a = 1$ , was shown to present a hidden chaotic attractor. In this work, we study the creation of this chaotic attractor, performing a bifurcation analysis by varying the parameter  $a$ . More specifically, we prove that, for  $a = 0$ , the Sprott A system has a line of equilibria in the  $z$ -axis, the phase space is foliated by concentric invariant spheres with two equilibrium points located at the south and north poles and each one of these spheres is filled by heteroclinic orbits of south pole–north pole type. For  $a \neq 0$ , the spheres are no longer invariant algebraic surfaces and the heteroclinic orbits are destroyed. We do a detailed numerical study for  $a > 0$  small, showing that small nested invariant tori and a *limit set*, which encompasses these tori and is the  $\alpha$ - and  $\omega$ -limit set of almost all orbits in the phase space, are formed in a neighborhood of the origin. As the parameter  $a$  increases, this *limit set* evolves into a hidden chaotic attractor, which coexists with the nested invariant tori. In particular, we find hidden chaotic attractors even for  $a < 1$ . Furthermore, we make a global analysis of Sprott A system, including the dynamics at infinity via the Poincaré compactification, showing that for  $a > 0$  the only orbit which escapes to infinity is the one contained in the  $z$ -axis, all other orbits are either homoclinic to a limit set (or to a hidden chaotic attractor, depending on the value of  $a$ ) or are contained on an invariant torus, depending on the initial condition considered.

Joint work with Alisson C. Reinol (Universidade Estadual de Maringá).

**Keywords:** *Sprott A system, hidden chaotic attractor, invariant algebraic surfaces, nested invariant tori, homoclinic and heteroclinic orbits.*



## A new chaotic bridge system

Marcos C. Mota

Instituto de Ciências Matemáticas e de Computação - Universidade de São Paulo  
[coutinhomotam@gmail.com](mailto:coutinhomotam@gmail.com)

**Abstract:** We present a study of dynamic aspects of the autonomous system

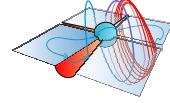
$$\dot{x} = yz, \quad \dot{y} = x - y, \quad \dot{z} = 1 - x(\alpha y + \beta x),$$

where  $(x, y, z) \in \mathbb{R}^3$  and  $\alpha, \beta \in [0, 1]$  are two parameters. It contains the so called Sprott B and the Sprott C systems (see [1]) at the two extremes of its parameter spectrum and we called it Sprott BC system. Such system can be considered as a *bridge system* which unify Sprott B and Sprott C systems (see [3] and [6], for instance). Here we present the complete description of its singularities and according to [5] we show that this system passes through a Hopf bifurcation at  $\alpha = 0$  and we compute the respective first Lyapunov coefficient. Using Poincaré compactification of a polynomial vector field in  $\mathbb{R}^3$  (see Appendix A of [2] for more details) we give a complete description of its dynamic on the Poincaré sphere at infinity. Finally we state some results about the non integrability of Sprott BC system.

Joint work with Regilene Oliveira (ICMC/USP).

## References

- [1] J. C. Sprott. *Some simple chaotic flows*, Phys. Rev. E **50** (2), 1994, R647–R650.
- [2] J. Llibre and X. Zhang. *Darboux theory of integrability for polynomial vector fields in  $\mathbb{R}^n$  taking into account the multiplicity at infinity*, Bull. Sci. Math., **133** (7), 2009, 765–778.
- [3] J. Lü, et al. *Bridge the gap between the Lorenz system and the Chen system*, Int. J. Bifurcat. Chaos. **12** (2002), 2917–2926.
- [4] M. C. Mota and R. D. S. Oliveira. *Dynamic Aspects of Sprott BC Chaotic System*, preprint, 2019.
- [5] Y. A. Kuznetsov. *Elements of applied bifurcation theory*, Vol. 112, Springer Science & Business Media, 2013.
- [6] Z. Elhadj and C. J. Sprott. *The unified chaotic system describing the Lorenz and Chua systems*, Facta Universitatis-Ser.: Elec. Energ. **3** (2010), 345–355.



# Existence of invariant sets in piecewise linear vector fields in $\mathbb{R}^3$ with sliding region

Velter, M. Q.

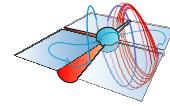
*Instituto de Matemática e Estatística - Universidade Federal de Goiás*  
*marianaqueirozvelter@gmail.com*

**Abstract:** The objective consists of the exploration of the linear vector fields in  $\mathbb{R}^3$ ,  $X(t, x)$ , separated by a plane (discontinuity surface) and we assume the existence of two parallel lines where the degree of contact of the fields is two (see [2]). In this context, together with the Filippov's convention (see [1]), it allows us first to make the qualitative study of the sliding vector field associated with  $X$ ,  $X^s$ , and thus reach our main goal, which is to determine the existence of invariant sets, such as cylinders and periodic orbits and to determine the global dynamics of the vector field  $X(t, x)$ .

Joint work with Durval José Tonon (IME/UFG).

## References

- [1] Filippov, A. F. *Differential equations with discontinuous righthand sides*, vol. **18** of Mathematics and its Applications (Soviet Series), Kluwer Academic Publishers Group, Dordrecht, 1988.
- [2] Freitas, B.R.; Medrado, J.C. *On the existence of limit cycles and invariant surfaces of sewing piecewise linear differential systems on  $\mathbb{R}^3$* , submitted (*arxivlink* : <https://arxiv.org/abs/1708.07168v1>).



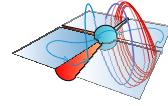
## Dinâmica global para uma classe de sistemas de Filippov

Maurício Lima

*Universidade Federal do ABC*

*limamauric@gmail.com*

**Resumo:** Consideraremos uma classe de sistemas suaves por partes cujas soluções estão associadas às chamadas soluções do tipo “peakon” e “multipeakon” para um sistema de equações parciais. Para essa classe de sistemas nosso objetivo é estudar o comportamento global e assintótico de suas soluções.



## Campos de vetores suaves por partes bidimensionais onde não vale a convenção de Filippov

Mayk Joaquim dos Santos

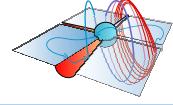
*Instituto de Matemática e Estatística - Universidade Federal de Goiás*

*maykjs@gmail.com*

**Resumo:** Apresentaremos uma possível classificação dos campos de vetores suaves por partes bidimensionais onde não vale a convenção de Filippov que são de codimensão 0 e 1, possuindo a cruz como descontinuidade, com o objetivo de determinar uma forma normal para cada caso, conforme foi feito nos trabalhos [1], [2], [3], [4], onde vale a convenção de Filippov.

## Referências

- [1] Cardoso, J. L.; Carvalho, T.; Tonon, D. J., *Canonical forms for codimension one planar piecewise smooth vector fields with sliding region*, Journal of Dynamics and Differential Equations, 30(4): 1899-1920, 2018.
- [2] Carvalho, T.; Tonon, D. J., *Normal forms for codimension one planar piecewise smooth fields*, Internat. J. Bifur. Chaos Appl. Sci. Engrg., 24(7): 1450090, 11, 2014.
- [3] Guardia, M.; Seara, T. M.; Teixeira, M. A., *Generic bifurcations of low codimension of planar Filippov systems*, J. Differential Equations, 250(4): 1967-2023, 2011.
- [4] Kuznetsov, Yu. A.; Rinaldi, S.; Gragnani, A., *One-parameter bifurcations in planar Filippov systems*, J. Internat. J. Bifur. Chaos Appl. Sci. Engrg., 13(8): 2157-2188, 2003.



## Estimadores de elemento de arco e de curvatura projetiva para curvas planas

Nathanni Vieira de Pádua

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*

*nathannipadua\_20@hotmail.com*

**Resumo:** Neste trabalho iremos estudar estimadores de elementos de arcos e de curvatura projetiva para curvas planas, quando as curvas são representadas por pontos e direções tangentes. A motivação para o estudo da geometria projetiva veio das belas artes. Foi em 1425 que o arquiteto italiano Brunelleschi começou a discutir a teoria geométrica da perspectiva, que foi consolidada em um tratado por Alberti alguns anos depois [2].

A geometria projetiva plana pode ser descrita como o estudo de propriedades geométricas que não são alteradas pela “projeção central”, ou seja, permanecem invariantes sob todas as transformações projetivas do espaço no qual se encontra. Alguns exemplos de invariantes projetivos são a retidão das linhas, a relação de posição de pontos e retas, e a razão cruzada de quatro pontos em uma reta, ou feixe de 4 retas passando por um ponto [1] e [4].

Uma curva regular plana projetiva  $A(\sigma)$  parametrizada pelo elemento de arco projetivo  $\sigma$  satisfaz uma equação diferencial linear de terceira ordem

$$\frac{d^3 A}{d\sigma^3} + 2k(\sigma) \frac{dA}{d\sigma} + (1 + k'(\sigma))A = 0. \quad (1)$$

A função  $k(\sigma)$  definida pela equação (1) será chamada de *curvatura projetiva*.

Para um gráfico  $\gamma(t) = (t, h(t), 1)$  numa carta afim  $(x, y, 1)$  temos a equação diferencial linear de terceira ordem

$$\gamma''' - \frac{h'''}{h''}\gamma'' = 0.$$

Definindo  $-\frac{h'''}{h''} = \frac{3}{2}\xi'$ , temos que o elemento de arco projetivo é  $H = \frac{1}{4}\frac{\xi'''}{\xi}$ . A curvatura projetiva é dada por

$$k = \frac{\sqrt[3]{2}}{36} \frac{(28\xi''''^2\xi^2 - 8\xi''''\xi''\xi'\xi + 7\xi'^2\xi''^2 - 30\xi\xi''\xi''^2)}{\xi^{\frac{4}{3}}\xi''^{\frac{8}{3}}}.$$

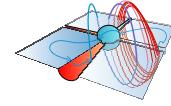
Observamos que  $H = 0$  (elemento de arco projetivo) para as cônicas e portanto a sua curvatura projetiva não está definida. Veja [3] e [4].

Na forma normalizada de (1) uma curva  $A(\sigma)$  de curvatura projetiva zero satisfaz a equação diferencial  $A'''(\sigma) + A(\sigma) = 0$ . Uma solução particular dessa equação diferencial é a espiral logarítmica.

Nesse trabalho iremos fazer uma abordagem dos trabalhos de Thomas Lewiner e Marcos Craizer [5, 6], apresentando o elemento de arco e curvatura baseados em *splines* projetivos, construídos por meio da adequação de espirais logarítmicas às amostras pontuais tangentes.

## Referências

- [1] D. Brannan, M. Esplen, and J. Gray. *Geometry*. Cambridge University Press, 2011.
- [2] H. Coxeter. *Projective Geometry*, Second Edition, University of Toronto Press, 1974.
- [3] R. Garcia; M. J. D. Carneiro. *Teorema dos quatro vértices e a sua recíproca*. XXXII Colóquio Brasileiro de Matemática. IMPA. 2019. Em preparação.
- [4] E. Lane. *Projective Differential Geometry Of Curves And Surfaces*, The University of Chicago Press Chicago, Illinois, U.S.A, 1932.
- [5] T. Lewiner and M. Craizer. Projective estimators for point-tangent representations of planar curves. In *Sibgrapi 2008 (XXI Brazilian Symposium on Computer Graphics and Image Processing)*, pages 223–229, Campo Grande, MT, october 2008. IEEE.
- [6] T. Lewiner and M. Craizer. *Projective splines and estimators for planar curves*. Journal of Mathematical Imaging and Vision, 36(1):81, 2010.



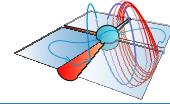
## Gradient systems of harmonic polynomials

Oscar A. Ramirez

*Universidade Federal de Viçosa*  
*oscar.ramirez@ufv.br*

**Abstract:** We characterize all local phase-portraits of the finite and infinite singular points of the gradient systems defined by the real harmonic polynomials in two variables.

We classify all the non-equivalent topological phase portraits of the gradient systems in the Poincaré disc defined by harmonic polynomials of degree less than five.



## Sobre sistemas com impasse e o estudo de fluxos em superfícies invariantes

Otávio Henrique Perez

*Instituto de Biociências, Letras e Ciências Exatas - Universidade Estadual Paulista*  
*otavio\_perez@hotmail.com*

**Resumo:** Sistemas com impasse são conhecidos na literatura por generalizar sistemas de equações diferenciais ordinárias (EDO's), além de modelar diversos problemas físicos, principalmente em engenharia elétrica. Mais precisamente, um sistema com impasse é escrito na forma

$$A(x)\dot{x} = F(x),$$

onde  $x \in \mathbb{R}^n$ ,  $A(x)$  é uma matriz  $n \times n$  e  $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$  é uma aplicação diferenciável. Este tipo de sistema é usado para estudar a dinâmica sobre a variedade lenta de um problema de perturbação singular. Neste trabalho, vamos usar esta classe de sistemas para estudar o fluxo de sistemas suaves em superfícies invariantes. Em particular, vamos estudar o fluxo de sistemas suaves sobre superfícies da forma

$$M = \{(x, y, z) \in \mathbb{R}^3 | f(x, y)z - g(x, y) = 0\},$$

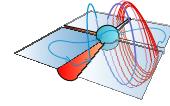
onde  $f, g : \mathbb{R}^2 \rightarrow \mathbb{R}$  são aplicações suaves. Superfícies da forma  $M$  podem ser conexas, desconexas, suaves, singulares, etc. Além disso, alguns sistemas famosos na literatura (Equação de Falkner-Skan, Sistema de Lorenz, Sistema de Chen) admitem superfícies dessa forma como superfícies invariantes, e é justamente nesses exemplos que aplicaremos os resultados obtidos.

Este é um trabalho em colaboração com Paulo Ricardo da Silva (IBILCE/UNESP). Este trabalho é financiado pela FAPESP (processo 2016/22310-0).

## Referências

- [1] J. Llibre, M. Messias e P. R. Silva. *Global dynamics of the Lorenz system with invariant algebraic surfaces*. International Journal of Bifurcation and Chaos, vol. 20, n. 10, 3137-3155, 2010.
- [2] J. Llibre, M. Messias e P. R. Silva. *Global dynamics in the Poincaré ball of the Chen system having invariant algebraic surfaces*. International Journal of Bifurcation and Chaos, vol. 22, 1250154, 2012.
- [3] J. Llibre, J. Sotomayor e M. Zhitomirskii. *Impasse bifurcations of constrained systems*. in Fields Inst. Commun., Lisbon, 2000, American Mathematical Society, Providence, RI, 2002.

- [4] J. Sotomayor e M. Zhitomirskii. *Impasse singularities of differential systems of the form  $A(x)x' = F(x)$* . Journal of Differential Equations 169 (2001), 567-587
- [5] M. Zhitomirskii. *Local normal forms for constrained systems on 2-manifolds*. Boletim da Sociedade Brasileira de Matematica 24 (1993), 211-232.



## The fundamental symmetry of homogeneous binary differential equations

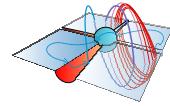
Tempesta, P.

*Universidade Federal de São João del-Rei*

*tempesta@ufs.br*

**Abstract:** Homogeneous binary differential equations are implicit differential equations of the form  $a(x, y)dy^2 + 2b(x, y)dxdy + c(x, y)dx^2 = 0$ , where the coefficients  $a, b, c$  are polynomial functions of degree  $n$ . This special class of binary differential equations has the property that the symmetry group is always non trivial. The existence of a fundamental symmetry in the group allow us to know when the two foliations have distinct shapes or not. Moreover, we can see that the invariant straight lines that can occur on the configurations have different behavior depending on parity of the degree of the coefficients.

Joint work with M. Manoel.

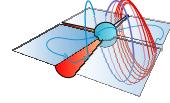


# Os teoremas da Variedade Estável e de Grobman-Hartman

Paulo Santana

*Instituto de Biociências, Letras e Ciências Exatas - Universidade Estadual Paulista*  
*santana.phr@gmail.com*

**Resumo:** O objetivo deste pôster é apresentar dois importantes resultados da Teoria Qualitativa das Equações Diferenciais Ordinárias: O Teorema da Variedade Estável e o Teorema de Grobman-Hartman.



# Boundary equilibrium bifurcations in a DC-DC buck power converter feeding a load of CPL-type

Rony Cristiano

*Instituto de Matemática e Estatística - Universidade Federal de Goiás*

*rony.cristiano@ufsc.br*

**Abstract:** In typical dc distribution systems with a cascaded converter architecture, loads connected to the bus by an electronic converter behave as constant power drawn from the feeder, and can be modelled as a constant power load (CPL); see [10]. This work considers a 3D-Filippov system that models the voltage control process at the output of the dc-dc bidirectional buck converter by feeding a CPL. The so-called *Boundary Equilibrium Bifurcations* (BEBs) are characterized by the collision of a regular equilibrium point of system with its discontinuity manifold; see [3]. Such BEBs are unique bifurcations of piecewise-smooth dynamical systems and, in recent years, have motivated several studies of their unfoldings (see [5, 6, 7, 8]), which was started by Filippov [4]. However, most of the works have considered two-dimensional systems and there are few studies related to BEBs in three-dimensional systems; see [2, 9].

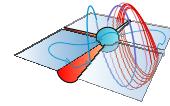
The goal of this work is to present some of the results on the BEBs in  $\mathbb{R}^3$  obtained in [1] [Chapter 7, Section 7.7] from an applied study performed with interconnected power converters and under a sliding mode control strategy. These results provide the conditions on the system parameters for the occurrence of BEBs, for the *persistence* and *nonsmooth fold* cases. In addition, from numerical simulations are observed a *boundary focus bifurcation* and the birth of a limit cycle with a sliding orbit segment. Other bifurcations of equilibria (saddle-node, Hopf and sliding Hopf) are also discussed in this work.

## References

- [1] R. Cristiano. *Bifurcations Analysis in Discontinuous Piecewise-Smooth Systems: Applications in Power Electronics*. PhD thesis, Federal University of Santa Catarina, Postgraduate Program in Automation Engineering and Systems, Brazil, 09 2018.
- [2] R. Cristiano, D. J. Pagano, L. Benadero, and E. Ponce. *Bifurcation analysis of a DC-DC bidirectional power converter operating with constant power load*. International Journal of Bifurcation and Chaos, 26(4):1630010 (18 pages), 2016.
- [3] M. Di Bernardo and S. J. Hogan. *Discontinuity-induced bifurcations of piecewise smooth dynamical systems*. Philosophical Transactions of the Royal Society of

London A: Mathematical, Physical and Engineering Sciences, 368(1930):4915–4935, 2010.

- [4] A. F. Filippov. *Differential Equations with Discontinuous Righthand Sides*. Kluwer Academic Publishers, Dordrecht, 1988.
- [5] P. A. Glendinning. *Classification of boundary equilibrium bifurcations in planar Filippov systems*. Chaos: An Interdisciplinary Journal of Nonlinear Science, 26(1):013108, 2016.
- [6] S. J. Hogan, M. E. Homer, M. R. Jeffrey, and R. Szalai. *Piecewise smooth dynamical systems theory: The case of the missing boundary equilibrium bifurcations*. Journal of Nonlinear Science, 26(5):1161–1173, 2016.
- [7] Y. A. Kuznetsov, S. Rinaldi, and A. Gragnani. *One-parameter bifurcations in planar Filippov systems*. International Journal of Bifurcation and Chaos, 13(8):2157–2188, 2003.
- [8] D. J. Pagano, E. Ponce, and F. Torres. *On double boundary equilibrium bifurcations in piecewise smooth planar systems*. Qualitative Theory of Dynamical Systems, 10(2):277–301, 2011.
- [9] D. J. W. Simpson. *A general framework for boundary equilibrium bifurcations of Filippov systems*. Chaos: An Interdisciplinary Journal of Nonlinear Science, 28(10):103114, 2018.
- [10] A. P. N. Tahim, D. J. Pagano, M. L. Heldwein, and E. Ponce. *Control of interconnected power electronic converters in dc distribution systems*. In XI Brazilian Power Electronics Conference, pages 269–274, Sept 2011.



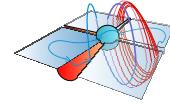
## Aplicação de campos de vetores suaves por partes na modelagem de câncer e HIV

Carvalho, T.

*Universidade de São Paulo - Ribeirão Preto*

*tiagocarvalho@usp.br*

**Resumo:** Iremos estudar alguns modelos de evolução de células cancerosas e também de evolução do vírus HIV utilizando campos vetoriais suaves por partes. Destaca-se a ocorrências de singularidades típicas, tais como dobra-dobra e cúspide-dobra.



## Existência de centros em sistemas diferenciais planares simétricos

Wilker Fernandes

*Universidade Federal de São João del-Rei*

*wilker@uvsj.edu.br*

**Resumo:** Investigamos condições para garantir a existência de centros em sistemas diferenciais planares cúbicos possuindo diferentes tipos de simetrias. Primeiramente encontramos as formas normais para tais sistemas usando quatro tipos diferentes de simetrias.

Em seguida, exibimos as condições necessárias e suficientes para a existência simultânea de dois centros para tais sistemas e também as condições necessárias e suficientes para a isocronicidade de tais centros.

Este é um trabalho em conjunto com Maša Dukarić (CAMTP, Maribor, Eslovênia) e Regilene Oliveira (ICMC-USP, São Carlos, Brasil).